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Sec 111

Professor Hutchings Midterm #2 (11/13/03)

6) Calculate $\int_C \vec{F}\cdot d\vec{r}$, where C is the unit circle oriented counterclockwise, and F is the following vector field in the plane: $\vec{F} = \langle -y^3 + \sin(\sin x),\ x^3 + \sin(\sin y)\rangle$

$C: \vec{r}(t) = \langle \cos t, \sin t \rangle, \quad 0 \le t \le 2\pi$

$\vec{r}\,'(t) = \langle -\sin t, \cos t \rangle$

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b f(\vec{r}(t))\cdot \vec{r}\,'(t)\, dt = \int_0^{2\pi} \langle -\sin^3 t + \sin(\sin(\cos t)),\ \cos^3 t + \sin(\sin(\sin t))\rangle \cdot \langle -\sin t, \cos t\rangle\, dt$$

$$= \int_0^{2\pi} \sin^4 t - (\sin t)(\sin(\sin(\cos t))) + \cos^4 t + (\cos t)(\sin(\sin(\sin t)))\, dt$$

$$= \int_0^{2\pi} \sin^4 t + \cos^4 t\, dt - \int_0^{2\pi} (\sin t)(\sin(\sin(\cos t)))\, dt + \int_0^{2\pi} (\cos t)(\sin(\sin(\sin t)))\, dt$$

$$\int_0^{2\pi} \sin^4 t + \cos^4 t\, dt = \int_0^{2\pi} \left(\frac{1-\cos(2t)}{2}\right)^2 + \left(\frac{1+\cos(2t)}{2}\right)^2 dt$$

$$= \frac{1}{4}\int_0^{2\pi} 1 \cancel{- 2\cos(2t)} + \cos^2(2t) + 1 \cancel{+ 2\cos(2t)} + \cos^2(2t)\, dt$$

$$= \frac{1}{4}\int_0^{2\pi} 2 + 2\cos^2(2t)\, dt = \frac{1}{4}\int_0^{2\pi} 2 + (1 + \cos(4t))\, dt$$

$$= \frac{1}{4}\int_0^{2\pi} 3 + \cos 4t\, dt = \frac{1}{4}\left[\left(3t + \frac{\sin 4t}{4}\right)\Big|_{t=0}^{t=2\pi}\right] = \frac{1}{4}(6\pi) = \frac{3\pi}{2}$$

let $u = \cos t$, $du = -\sin t\, dt$

$$-\int_0^{2\pi} (\sin t)(\sin(\sin(\cos t)))\, dt = \int_1^1 \sin(\sin u)\, du = 0 \quad \text{(bcs limits are same)}$$

$u = \sin t$, $du = \cos t\, dt$

$$\int_0^{2\pi} (\cos t)(\sin(\sin(\sin t)))\, dt = \int_0^0 \sin(\sin u)\, du = 0$$

$$\therefore \int_C \vec{F}\cdot d\vec{r} = \frac{3\pi}{2} + 0 + 0 = \boxed{\frac{3\pi}{2}}$$